තුන්වන මාර පරීක්ෂණය - 2017

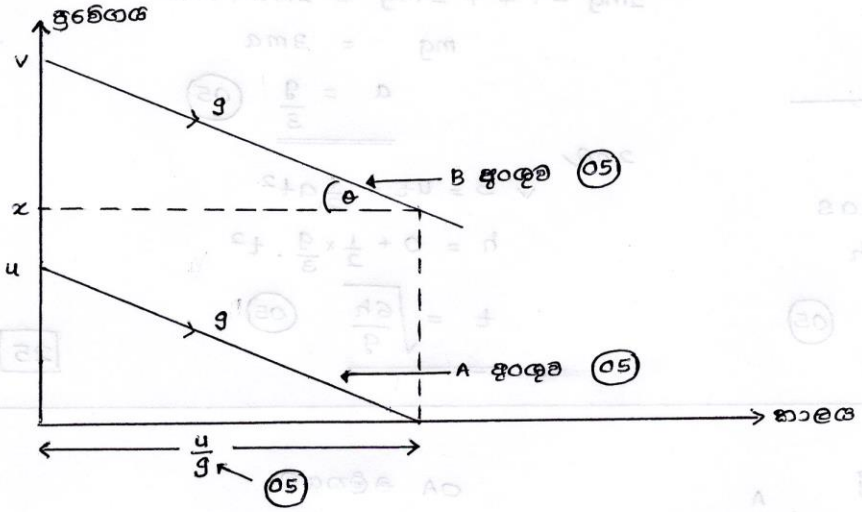
12 ශ්‍රේණිය

සංයුක්ත ගණිතය - II

ලකුණු දීමේ පටිපාටිය.

A කොටස

(1)



ඇවරණය = Tan θ

$g = \frac{v-x}{u/g}$

$u = v-x$

$x = v-u$ — (05)

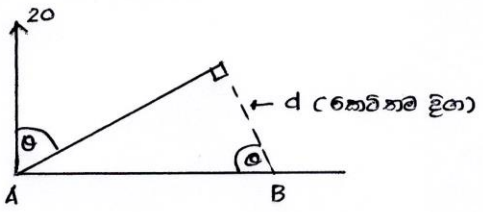
දුර = $\frac{1}{2}(v+x)\times\frac{u}{g}$

$= \frac{1}{2}(v+v-u)\times\frac{u}{g}$

$= \frac{(2v-u).u}{2g}$ — (05)

25

(2)



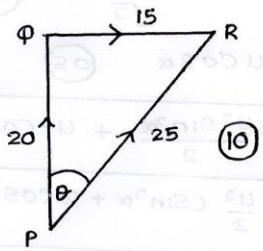
$\underline{v}$ (නෑ, සෝ) = $\underline{v}$ (නෑ, සොං) + $\underline{v}$ (සොං, සෝ)

= ↑20 + ←15 (05)

$\overrightarrow{PR} = \overrightarrow{P\Phi} + \overrightarrow{\Phi R}$

$\underline{v}$ (නෑ, සොං) = ↑ 20

$\underline{v}$ (සෝ, සොං) = ← 15



(10)

කෙටිනම දිග = 20 Cos θ

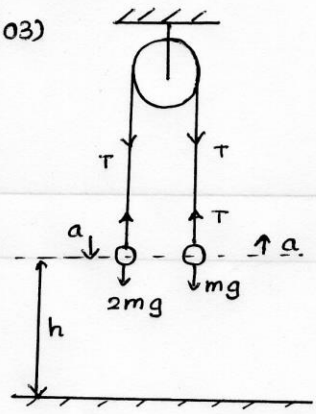
= $20\times\frac{20}{25}$ (05)

= 16 km (05)

කාලය = $\frac{20\ \text{Sin}\theta}{25} \Rightarrow \frac{20}{25}\times\frac{15}{25} \Rightarrow \frac{12}{25}$ h

25

(03)



2m ස්කන්ධය $\downarrow F = ma$ යෙදීම

$2mg - T = 2ma \rightarrow ①$ (05)

m ස්කන්ධය $\uparrow F = ma$

$T - mg = ma \rightarrow ②$ (05)

① + ②

$2mg - T + T - mg = 2ma + ma$

$mg = 3ma$

$a = \frac{g}{3}$ (05)

2m ස්කන්ධය

$\downarrow v^2 = u^2 + 2as$

$v^2 = \frac{2g}{3} \times h$

$v = \sqrt{\frac{2gh}{3}}$ (05)

2m ස්කන්ධය

$\downarrow s = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2} \times \frac{g}{3} \cdot t^2$

$t = \sqrt{\frac{6h}{g}}$ (05)

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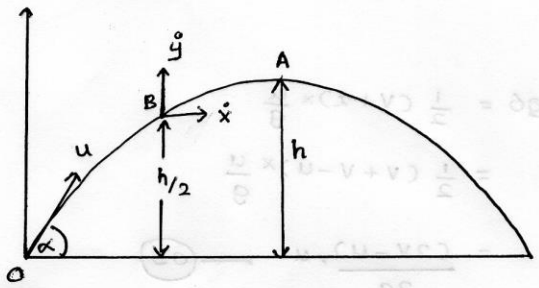
(04)



OA ඔළුතලය

$\uparrow v^2 = u^2 + 2as$

$0 = (u\sin\alpha)^2 - 2gh$

$h = \frac{u^2\sin^2\alpha}{2g}$ (05)

OB ඔළුතලය

ස්කන්ධය $\uparrow v^2 = u^2 + 2as$

$\dot{y}^2 = (u\sin\alpha)^2 - 2g \times \frac{h}{2}$

$\dot{y}^2 = u^2\sin^2\alpha - g \times \frac{u^2\sin^2\alpha}{2g}$

$\dot{y} = \frac{u\sin\alpha}{\sqrt{2}}$ (05)

$\dot{x} = u\cos\alpha$ (05)

$\bar{v} = \sqrt{\frac{u^2\sin^2\alpha}{2} + u^2\cos^2\alpha}$ (05)

$\bar{v} = \sqrt{\frac{u^2}{2}(\sin^2\alpha + 2\cos^2\alpha)}$

$\bar{v} = \sqrt{\frac{u^2}{2}(\sin^2\alpha + \cos^2\alpha + \cos^2\alpha)}$

$\bar{v} = \frac{u}{\sqrt{2}}(1 + \cos^2\alpha)^{1/2}$ (05)

25

(05) $\overrightarrow{OA} = \underline{a} + 2\underline{b}$ $\overrightarrow{OB} = 3\underline{a} - \underline{b}$

$OA \perp OB$ නම්,

$\overrightarrow{OA} \cdot \overrightarrow{OB} = 0 \rightarrow (05)$

$(\underline{a} + 2\underline{b}) \cdot (3\underline{a} - \underline{b}) = 0$

$3|\underline{a}|^2 + 5\,\underline{a} \cdot \underline{b} - 2|\underline{b}|^2 = 0 \rightarrow (05)$ [$\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$ නිසා]

$\underline{a} \cdot \underline{b} = \frac{2}{5}|\underline{b}|^2 - \frac{3}{5}|\underline{a}|^2 \rightarrow (05)$

$|\underline{a}| = 2$ සහ $|\underline{b}| = 1$ නම්,

$\underline{a} \cdot \underline{b} = \frac{2}{5} - \frac{12}{5} = -2$

$|\underline{a}||\underline{b}| \cos\theta = -2 \rightarrow (05)$ මෙහි θ යනු a සහ b අතර කෝණය.

$0 \le \theta \le \pi$

$2\cos\theta = -2$

$\cos\theta = -1$

$\theta = \pi$ (05)

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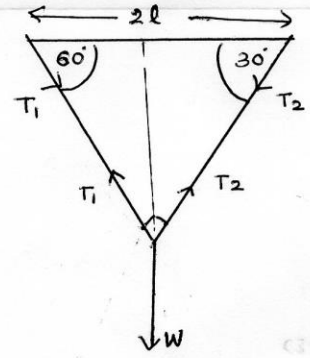
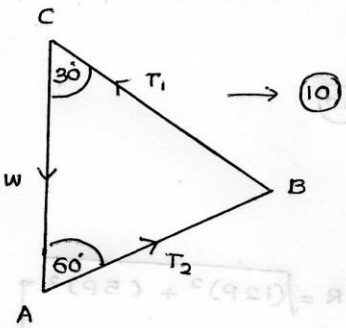
(06)



කෝණවලට සහ බල ලකුණු කිරීමට (05)



$\frac{W}{AC} = \frac{T_2}{AB} = \frac{T_1}{BC}$ → (10)

$T_1 = W \operatorname{Sec} 30^\circ$

$T_1 = W \cdot \frac{2}{\sqrt{3}}$

$T_1 = \frac{2W}{\sqrt{3}} \rightarrow (05)$

$T_2 = W \operatorname{Sec} 60^\circ$

$T_2 = W \times 2$

$T_2 = 2W \rightarrow (05)$

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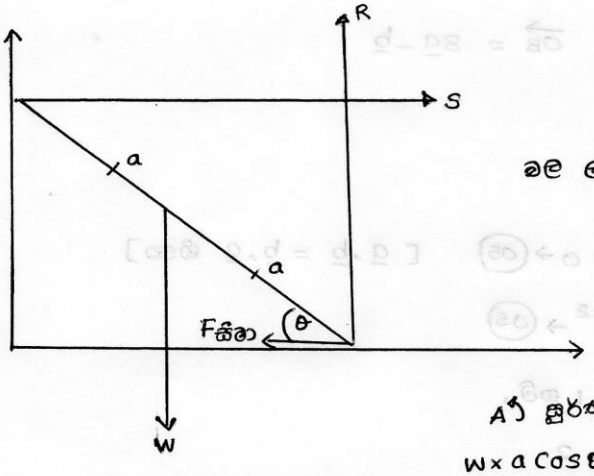
(07)



බල ලකුණු කිරීමට (05)

$F_{සීමා} = \mu R$

$\uparrow \; R - W = 0 \longrightarrow (1)$ (05)

$R = W$

$\rightarrow \; S - F_{සීමා} = 0$

$S - \mu R = 0$

$S = \mu W \longrightarrow (2)$ (05)

A ↺ සුර්ණ

$W \times a\cos\theta - S \times 2a\sin\theta = 0 \rightarrow (3)$ (05)

$S = \dfrac{W\cos\theta}{2\sin\theta}$

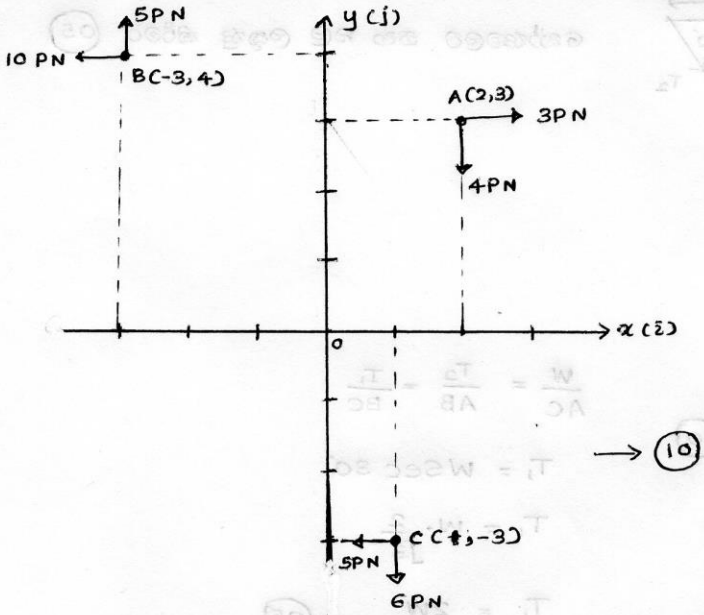
$S = \dfrac{W}{2}\cot\theta$

$\mu W = \dfrac{W}{2}\cot\theta$

$\underline{\underline{2\mu = \cot\theta}} \rightarrow$ (05)

25

(08)



$\longrightarrow$ (10)

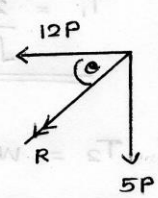
බල විභේදනයෙන්,

$\uparrow \; y = 5P - 4P - 6P$

$\underline{\underline{y = -5PN}}$ (05)

$\rightarrow \; x = 3P - 10P - 5P$

$\underline{\underline{x = -12PN}}$ (05)



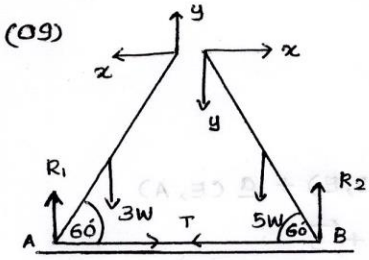
$R = \sqrt{(12P)^2 + (5P)^2}$

$R = \underline{\underline{13PN}}$

$\theta = \underline{\underline{\tan^{-1}(5/12)}}$ (05)

25

(09)



දණ්ඩක දිග 2a ලෙස ගනිමු.

AB දණ්ඩට A හරහා

$3W \times a\cos 60^\circ = x \times 2a\cos 30^\circ + y \times 2a\cos 60^\circ$ (05)

$\frac{3W}{2} = 2x \times \frac{\sqrt{3}}{2} + 2y \times \frac{1}{2}$

$\frac{3W}{2} = \sqrt{3}x + y \longrightarrow (1)$

CB දණ්ඩට C හරහා

$5W \times a\cos 60^\circ + y \times 2a\cos 60^\circ = x \times 2a\cos 30^\circ$ (05)

$5W \times \frac{1}{2} + 2y \times \frac{1}{2} = \frac{2\sqrt{3}}{2} \cdot x$

$\frac{5W}{2} + y = \sqrt{3}x \longrightarrow (2)$

(1) + (2)

$4W = 2\sqrt{3}x$

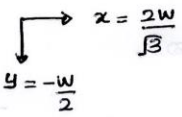
$x = \frac{2W}{\sqrt{3}} \rightarrow (05)$

(1) න්

$\frac{3W}{2} = \sqrt{3} \times \frac{2W}{\sqrt{3}} + y$

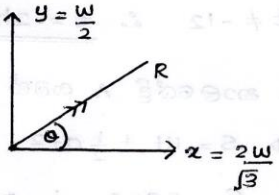
$y = -\frac{W}{2} \rightarrow (05)$

B හි ප්‍රතික්‍රියාව



$R^2 = \frac{W^2}{4} + \frac{4W^2}{3}$

$R = \frac{\sqrt{19}\,W}{2\sqrt{3}}$ ← (05) →

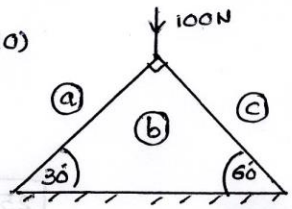


දිශාව, $\tan\theta = \frac{W/2}{2W/\sqrt{3}}$

$\theta = \tan^{-1}\left(\sqrt{3}/4\right)$

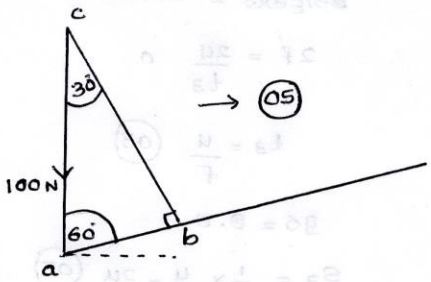
25

(10)



දණ්ඩ	බලය	විශාලත්වය (N)
AB	තෙරපුම (05)	50 → (05)
BC	තෙරපුම (05)	$50\sqrt{3}$ → (05)

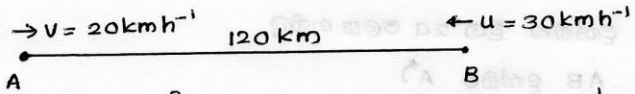
ප්‍රතනාබල සටහන



→ (05)

25

(11) (a)



$\underline{V}(B,A) = \underline{V}(B,E) + \underline{V}(E,A)$

$= \overleftarrow{30} + \overleftarrow{20}$

$= \overleftarrow{50}\ kmh^{-1}$ (10)

$\underline{a}(B,A) = \underline{a}(B,E) + \underline{a}(E,A)$

$= \overleftarrow{5} + \overleftarrow{5}$

$= 10\ kmh^{-2}$ (10)

A සහ B මුණ ගැසීමට ගතවන කාලය t නම්,

$\leftarrow\ s = ut + \frac{1}{2}at^2$

$120 = 50 \times t + \frac{1}{2} \times 10 \times t^2$ (10)

$t^2 + 10t - 24 = 0$

$(t+12)(t-2) = 0$

$t \neq -12 \quad \therefore \underline{t = 2h}$ (10)

t කාලයේදී A ගමන් කළ දුර s නම්,

$\rightarrow\ s = ut + \frac{1}{2}at^2$

$s = 20 \times 2 + \frac{1}{2} \times 5 \times 4$

$\underline{s = 50km}$ (15)

A හි ප්‍රවේගය V නම්

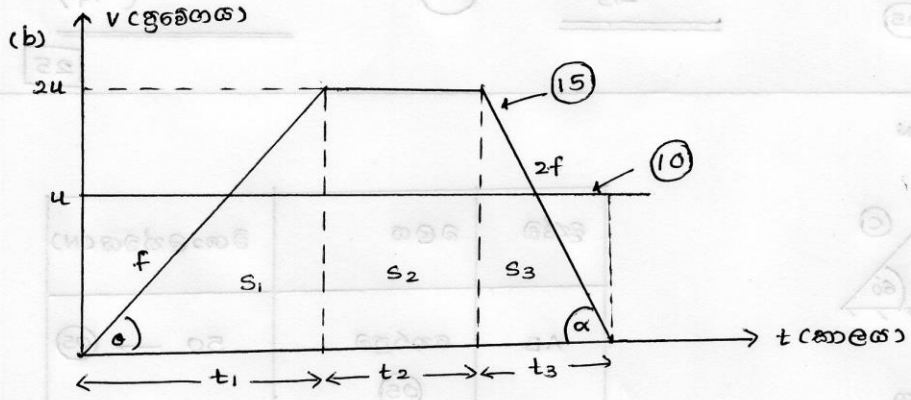
$\rightarrow\ V = u + at$

$V = 20 + 5 \times 2$

$\underline{V = 30\ kmh^{-1}}$ (10)

65

(b)



25

ඇල්ලුමය = Tanθ

$f = \frac{2u}{t_1}$

$t_1 = \frac{2u}{f}$ (05)

දුර = ව.ඒ.

$S_1 = \frac{1}{2} \times \frac{2u}{f} \times 2u$ (05)

$S_1 = \frac{2u^2}{f}$ (05)

$t_2 = \frac{u}{2f}$ (දිග්නය)

දුර = ව.ඒ

$S_2 = 2u \times \frac{u}{2f}$ (05)

$S_2 = \frac{u^2}{f}$ (05)

මන්දනය = Tanα

$2f = \frac{2u}{t_3}$

$t_3 = \frac{u}{f}$ (05)

දුර = ව.ඒ.

$S_3 = \frac{1}{2} \times \frac{u}{f} \times 2u$ (05)

$S_3 = \frac{u^2}{f}$ (05)

40

B ට A පසුකර යා නොහැකි නම්,

B ගමන් කරන දුර $-d \leq$ A ගමන් කරන දුර (05)

$s_1 + s_2 + s_3 - d \leq u(t_1 + t_2 + t_3)$ (05)

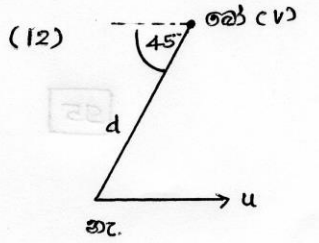
$$\frac{2u^2}{f} + \frac{u^2}{f} + \frac{u^2}{f} - d \leq u\left(\frac{2u}{f} + \frac{u}{2f} + \frac{u}{f}\right)$$

$$\frac{4u^2}{f} - d \leq \frac{7u^2}{2f}$$

$$\frac{u^2}{2f} \leq d \quad (10)$$

20

(12)



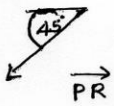
$\underline{V}$ (නැ, සෝ) $= \overrightarrow{u}$

$\underline{V}$ (බෝ, සෝ) $= V$ (10)

$\underline{V}$ (බෝ, නැ) $= 45°$

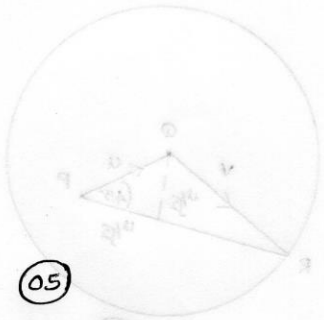
$\underline{V}$ (බෝ, නැ) $=$ $\underline{V}$ (බෝ, සෝ) + $\underline{V}$ (සෝ, නැ)

$= V + \overleftarrow{u}$

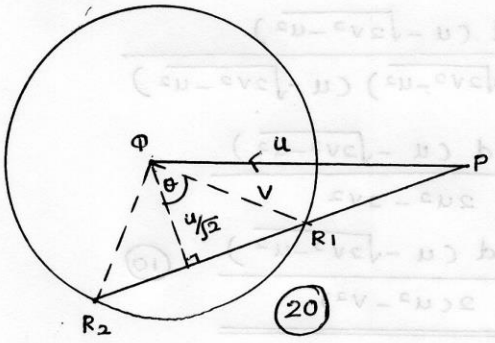


$= \overleftarrow{u} + V$ (10)

$\overrightarrow{PR} = \overrightarrow{P\phi} + \overrightarrow{\phi R}$



$V < u$ වන විට



20

$\cos\theta = \frac{u}{\sqrt{2}V}$ (05)

$\theta = \cos^{-1}\left(\frac{u}{\sqrt{2}V}\right)$ (05)

∴ දිශා අතර කෝණය $= 2\cos^{-1}\left(\frac{u}{\sqrt{2}V}\right)$

$$PR_1 = \frac{u}{\sqrt{2}} - \sqrt{V^2 - \frac{u^2}{2}}$$

$$= \frac{u - \sqrt{2V^2 - u^2}}{\sqrt{2}} \quad (10)$$

$$PR_2 = \frac{u + \sqrt{2V^2 - u^2}}{\sqrt{2}} \quad (10)$$

කාල අන්තරය $= \frac{d}{PR_1} - \frac{d}{PR_2}$ (05)

$= \frac{d}{\frac{u-\sqrt{2v^2-u^2}}{\sqrt{2}}} - \frac{d}{\frac{u+\sqrt{2v^2-u^2}}{\sqrt{2}}}$ (05)

$= \frac{\sqrt{2}\,d}{u-\sqrt{2v^2-u^2}} - \frac{\sqrt{2}\,d}{u+\sqrt{2v^2-u^2}}$

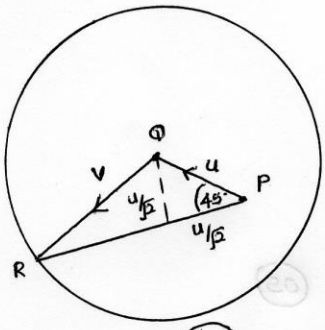
$= \frac{\sqrt{2}\,d\,(u+\sqrt{2v^2-u^2} - u+\sqrt{2v^2-u^2})}{u^2-(2v^2-u^2)}$

$= \frac{\sqrt{2}\,d.\ 2\times\sqrt{2v^2-u^2}}{2(u^2-v^2)}$

$= \frac{d\sqrt{4v^2-2u^2}}{(u^2-v^2)}$ (15)

95

$V > u$ නම් විට,



(15)

$PR = \frac{u}{\sqrt{2}} + \sqrt{v^2 - \frac{u^2}{2}}$

$= \frac{u+\sqrt{2v^2-u^2}}{\sqrt{2}}$ (10)

කාලය $= \frac{d}{PR}$ (05)

$= \frac{\sqrt{2}\,d}{u+\sqrt{2v^2-u^2}}$ (05)

$= \frac{\sqrt{2}\,d\,(u-\sqrt{2v^2-u^2})}{(u+\sqrt{2v^2-u^2})(u-\sqrt{2v^2-u^2})}$

$= \frac{\sqrt{2}\,d\,(u-\sqrt{2v^2-u^2})}{2u^2-2v^2}$

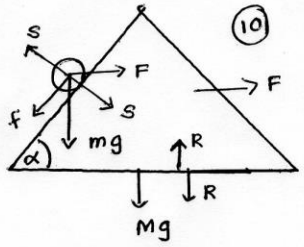
$= \frac{\sqrt{2}\,d\,(u-\sqrt{2v^2-u^2})}{2(u^2-v^2)}$ (10)

මෙම කාලය තුළ නැව ගන දුර $= \frac{u\times\sqrt{2}\,d\,(u-\sqrt{2v^2-u^2})}{2(u^2-v^2)}$

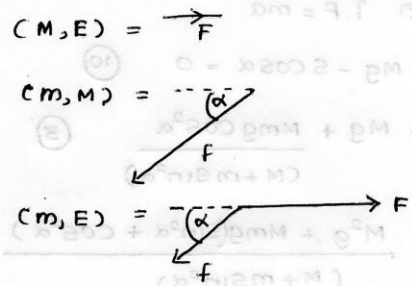
$= \frac{\sqrt{2}\,ud\,(u-\sqrt{2v^2-u^2})}{2(u^2-v^2)}$ (10)

55

(13)



(10)



(10)

(i) (m) අංශුවට $\swarrow$ $F = ma$

$$mg\sin\alpha = m(f - F\cos\alpha) \longrightarrow (1) \quad (15)$$

පද්ධතියට $\longrightarrow$ $F = ma$

$$0 = m(F - f\cos\alpha) + MF \longrightarrow (2) \quad (15)$$

$\boxed{50}$

(1) $\times \cos\alpha$ + (2)

$$F(M + m - m\cos^2\alpha) = mg\sin\alpha\cos\alpha$$

$$F = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha} \quad (10)$$

(ii) (1) ෆ්,

$$mg\sin\alpha = m\left(f - \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}\right)$$

$$g\sin\alpha + \frac{mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha} = f$$

$$\frac{Mg\sin\alpha + mg\sin^3\alpha + mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha} = f \quad (05)$$

$$\frac{Mg\sin\alpha + mg\sin\alpha(\sin^2\alpha + \cos^2\alpha)}{M + m\sin^2\alpha} = f$$

$$f = \frac{(M + m)g\sin\alpha}{M + m\sin^2\alpha} \quad (10)$$

$\boxed{25}$

(iii) (m) අංශුවට $\nwarrow$ $F = ma$

$$S - mg\cos\alpha = -mF\sin\alpha \quad (10)$$

$$S = -m\sin\alpha\left[\frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}\right] + mg\cos\alpha \quad (05)$$

$$S = \frac{-m^2g\sin^2\alpha\cos\alpha + m^2g\sin^2\alpha\cos\alpha + mMg\cos\alpha}{M + m\sin^2\alpha} \quad (05)$$

$$S = \frac{Mmg\cos\alpha}{(M + m\sin^2\alpha)} \quad (10)$$

$\boxed{30}$

(iv) කුඤ්ඤයට $\uparrow F = ma$

$$R - Mg - S\cos\alpha = 0 \quad (10)$$

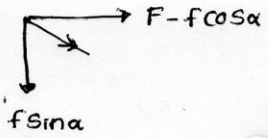
$$R = Mg + \frac{Mmg\cos^2\alpha}{(M + m\sin^2\alpha)} \quad (5)$$

$$R = \frac{M^2g + Mmg(\sin^2\alpha + \cos^2\alpha)}{(M + m\sin^2\alpha)}$$

$$R = \frac{Mg(M+m)}{(M + m\sin^2\alpha)} \quad (05)$$

$\boxed{20}$

(v) දුංගුවේ ත්වරණය = a



$$a = \sqrt{(F - f\cos\alpha)^2 + f^2\sin^2\alpha} \quad (10)$$

$$a = \sqrt{\left(\frac{-Mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}\right)^2 + \frac{(M+m)^2g^2\sin^2\alpha.\sin^2\alpha}{(M + m\sin^2\alpha)^2}} \quad (05)$$

$$= \frac{g\sin\alpha}{M + m\sin^2\alpha}\sqrt{M^2\cos^2\alpha + (M+m)^2\sin^2\alpha}$$

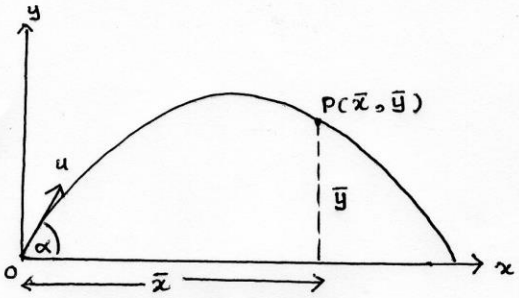
$$= \frac{g\sin\alpha}{M + m\sin^2\alpha}\sqrt{M^2(\cos^2\alpha + \sin^2\alpha) + m^2\sin^2\alpha + 2Mm\sin^2\alpha}$$

$$= \frac{g\sin\alpha}{M + m\sin^2\alpha}\sqrt{M^2 + m^2\sin^2\alpha + 2Mm\sin^2\alpha}$$

$$= \frac{g\sin\alpha}{M + m\sin^2\alpha}\sqrt{M^2 + m\sin^2\alpha(m + 2M)} \quad (10)$$

$\boxed{25}$

(14)



OP දිශාවෙන්

$\rightarrow s = ut + \frac{1}{2}at^2$ යෙදීම

$\bar{x} = u\cos\alpha . t$ (10)

$t = \frac{\bar{x}}{u\cos\alpha}$

OP දිශාවෙන්

$\uparrow s = ut + \frac{1}{2}at^2$ යෙදීම

$\bar{y} = u\sin\alpha . \frac{\bar{x}}{u\cos\alpha} - \frac{1}{2}g\left(\frac{\bar{x}}{u\cos\alpha}\right)^2$ (10)

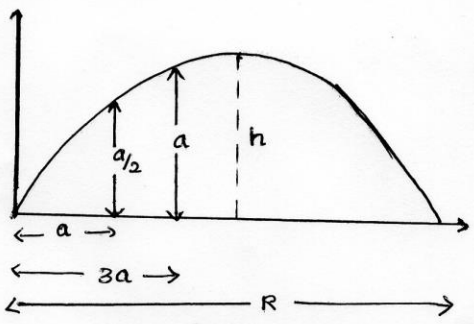
$\bar{y} = \bar{x}\tan\alpha - \frac{g\bar{x}^2}{2u^2\cos^2\alpha}$

$\bar{y} = \bar{x}\tan\alpha - \frac{g\bar{x}^2}{2u^2}\sec^2\alpha$ (10)

$\bar{x} \rightarrow x$ ලෙසද $\bar{y} \rightarrow y$ ලෙසද ගනිමු.

$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$ (5)

35



$x \rightarrow a$ වන විට $y \rightarrow \frac{a}{2}$ වේ.

$\frac{a}{2} = a\tan\alpha - \frac{ga^2}{2u^2}\sec^2\alpha$ (10)

$\frac{ga}{2u^2}\sec^2\alpha = \tan\alpha - \frac{1}{2} \rightarrow (1)$

$x \rightarrow 3a$ වන විට $y \rightarrow a$ වේ.

$a = 3a\tan\alpha - \frac{g(3a)^2}{2u^2}\sec^2\alpha$ (15)

$\frac{9ag}{2u^2}\sec^2\alpha = 3\tan\alpha - 1 \quad -(2)$

(1)/(2)

$\frac{\frac{ga^2}{2u^2}\sec^2\alpha}{\frac{9ga}{2u^2}\sec^2\alpha} = \frac{\tan\alpha - 1/2}{3\tan\alpha - 1}$ (10)

$3\tan\alpha - 1 = 9\tan\alpha - \frac{9}{2}$

$6\tan\alpha = 7/2$

$\tan\alpha = \frac{7}{12} \Rightarrow \alpha = \tan^{-1}(7/12)$ (15)

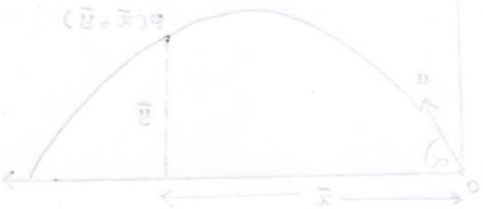
50

$x \to R$ වන විට $y \to 0$ වේ.

$$0 = R\tan\alpha - \frac{gR^2}{2u^2}\sec^2\alpha \quad (10)$$

$$\frac{gR^2}{2u^2}\sec^2\alpha = R\tan\alpha$$

$$\frac{gR^2}{2u^2}\sec^2\alpha = \tan\alpha \rightarrow (3)$$



$$(1)/(3) \quad \frac{\frac{ga}{2u^2}\sec^2\alpha}{\frac{gR}{2u^2}\sec^2\alpha} = \frac{\tan\alpha - 1/2}{\tan\alpha} \quad (10)$$

$$a\tan\alpha = R(\tan\alpha - 1/2)$$

$$R = \frac{a\tan\alpha}{\tan\alpha - 1/2}$$

$$R = \frac{a \times 7/12}{7/12 - 1/2}$$

$$\underline{\underline{R = 7a}} \quad (15)$$

$\boxed{35}$

උපරිම උස h ලෙස ගනිමු.

$x \to \frac{R}{2}$ විට $y \to h$ වේ.

$$h = \frac{R}{2}\tan\alpha - \frac{g(R/2)^2}{2u^2}\sec^2\alpha \quad (10)$$

$$h = \frac{R}{2}\tan\alpha - \frac{gR^2}{8u^2}\sec^2\alpha$$

$$\frac{gR^2}{8u^2}\sec^2\alpha = \frac{R}{2}\tan\alpha - h \rightarrow (4)$$

$$(1)/(4) \quad \frac{\frac{ga}{2u^2}\sec^2\alpha}{\frac{gR^2}{4\times 2u^2}\sec^2\alpha} = \frac{\tan\alpha - 1/2}{\frac{R}{2}\tan\alpha - h} \quad (10)$$

$$\frac{4a}{R^2} = \frac{\tan\alpha - 1/2}{R/2\,\tan\alpha - h}$$

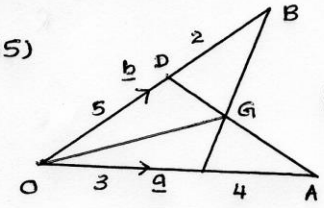
$$4a(R/2\,\tan\alpha - h) = R^2(\tan\alpha - 1/2)$$

$$4a\left(\frac{7a}{2}\times\frac{7}{12} - h\right) = 49a^2\left(\frac{7}{12} - \frac{1}{2}\right)$$

$$\frac{49a}{24} - h = \frac{49a}{48} \Rightarrow \underline{\underline{h = \frac{49a}{48}}} \quad (10)$$

$\boxed{30}$

(15)



(05)

$\overrightarrow{OD} = \frac{5}{7}\overrightarrow{OB} = \frac{5}{7}\underline{b}$

$\overrightarrow{OE} = \frac{3}{7}\overrightarrow{OA} = \frac{3}{7}\underline{a}$

$\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG}$ (05)

$= \underline{b} + \lambda\overrightarrow{BE}$

$= \underline{b} + \lambda(\overrightarrow{BO} + \overrightarrow{OE})$

$= \underline{b} + \lambda(-\underline{b} + \frac{3}{7}\overrightarrow{OA})$

$= \underline{b} + \lambda(-\underline{b} + \frac{3}{7}\underline{a})$

$\overrightarrow{OG} = \underline{b} + \lambda(\frac{3}{7}\underline{a} - \underline{b}) \rightarrow ①$

(10)

$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$

$= \underline{a} + M\overrightarrow{AD}$

$= \underline{a} + M(\overrightarrow{AO} + \overrightarrow{OD})$ (05)

$= \underline{a} + M(-\underline{a} + \frac{5}{7}\overrightarrow{OB})$

$= \underline{a} + M(-\underline{a} + \frac{5}{7}\underline{b})$

$\overrightarrow{OG} = \underline{a} + M(\frac{5}{7}\underline{b} - \underline{a}) \rightarrow ②$

(10)

① = ② $\quad \underline{b} + \lambda(\frac{3}{7}\underline{a} - \underline{b}) = \underline{a} + M(\frac{5}{7}\underline{b} - \underline{a})$ (05)

$(\frac{3\lambda}{7} - 1 + M)\underline{a} + (1 - \lambda - \frac{5M}{7})\underline{b} = \underline{0}$ (05)

$\underline{a}$ සහ $\underline{b}$ ඝාතන්තර නොවන නිසා,

$\frac{3\lambda}{7} - 1 + M = 0 \rightarrow ③$ (05)

$1 - \lambda - \frac{5M}{7} = \underline{0} \rightarrow ④$ (05)

③ + $\frac{3}{7}$ × ④ $\quad M(1 - \frac{15}{49}) = 1 - \frac{3}{7}$

$M = \frac{4 \times 7}{34}$

$M = \frac{14}{17}$ (05)

④ හි,

$\lambda = 1 - \frac{5 \times 14}{7 \times 17}$

$\lambda = 1 - \frac{10}{17}$

$\lambda = \frac{7}{17}$ (05)

$\overrightarrow{OG} = \underline{b} + \frac{7}{17}(\frac{3}{7}\underline{a} - \underline{b})$

$\overrightarrow{OG} = \underline{b} + \frac{3\underline{a}}{17} - \frac{7\underline{b}}{17}$

$\overrightarrow{OG} = \frac{1}{17}(3\underline{a} + 10\underline{b})$ (10)

75

(b) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

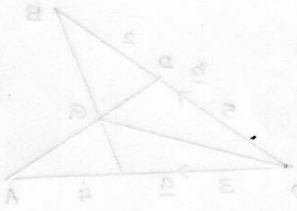
$= -2\underline{u} - 4\underline{v} + 3\underline{u} + 2\underline{v}$

$\overrightarrow{AB} = \underline{u} - 2\underline{v}$ (05)

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= -3\underline{u} - 2\underline{v} + 5\underline{u} + \lambda\underline{v}$

$\overrightarrow{BC} = 2\underline{u} + (\lambda - 2)\underline{v}$ (05)



A, B, C ඒකරේඛීය නිසා,

$\overrightarrow{AB} = M(\overrightarrow{BC})$ (05)

$\underline{u} - 2\underline{v} = M(2\underline{u} + (\lambda-2)\underline{v})$

$(1-2M)\underline{u} + (-2-M(\lambda-2))\underline{v} = \underline{0}$ (10)

$\underline{u}$ හා $\underline{v}$ සමාන්තර නොවන නිසා,

$1-2M = 0 \rightarrow (1)$

$M = \frac{1}{2}$ (05)

$-2 - M(\lambda-2) = 0$

$-2 - \frac{1}{2}(\lambda-2) = 0$

$-4 - \lambda + 2 = 0$

$\lambda = -2$ (05)

$AB : BC = 1 : 2$ (05)

$\overrightarrow{OD} = \sqrt{a^2+b^2} = \sqrt{5}$ (05) $\quad \overrightarrow{OA} = \sqrt{20}$

$\overrightarrow{OA} \cdot \overrightarrow{OD} = (2\underline{u} + 4\underline{v}) \cdot (a\underline{u} + b\underline{v}) = |\overrightarrow{OA}|\,|\overrightarrow{OD}| \cos\frac{\pi}{3}$ (10)

$2a\,\underline{u}\cdot\underline{u} + 4b\,\underline{v}\cdot\underline{v} + 2b\,\underline{u}\cdot\underline{v} + 4a\,\underline{u}\cdot\underline{v} = \sqrt{20} \times \sqrt{5} \times \frac{1}{2}$

$2a + 4b + 2b\,|\underline{u}|\,|\underline{v}| \cos 60^\circ + 4a\,|\underline{u}|\,|\underline{v}| \cos 60^\circ = 5$

$2a + 4\underline{b} + \underline{b} + 2a = 5$

$4a + 5b = 5 \rightarrow (1)$

(10)

(1) ෙන් $a = \frac{5}{4}(1-b)$

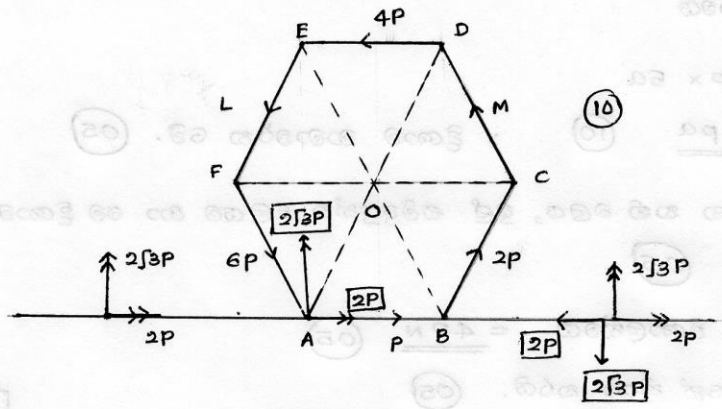
$a^2 + b^2 = 5 \rightarrow (2)$

$\frac{25}{16}(1-b)^2 + b^2 = 5$

$25 + 25b^2 - 50b + 16b^2 = 80$

$41b^2 - 50b - 55 = 0$ (10)

(16)



(10)

(i) $\overrightarrow{X} = P + 2P \times \frac{1}{2} - M \times \frac{1}{2} - 4P - L \times \frac{1}{2} + 6P \times \frac{1}{2} = 0$ (15)

$M + L = 2P \rightarrow ①$

$\uparrow Y = 2P \times \frac{\sqrt{3}}{2} + M \times \frac{\sqrt{3}}{2} - L \times \frac{\sqrt{3}}{2} - 6P \times \frac{\sqrt{3}}{2} = 0$ (15)

$M - L = 4P \rightarrow ②$

① සහ ② න්, $M = 3P\ N$ (05) $L = -P\ N$ (05)

පද්ධතියෙහි A හි

$G = 2P \times a \times \frac{\sqrt{3}}{2} + 3P \times 2a \times \frac{\sqrt{3}}{2} + 4P \times 2a \times \frac{\sqrt{3}}{2} - P \times a \times \frac{\sqrt{3}}{2}$ (15)

$G = \frac{15\sqrt{3}\ Pa}{2}$ (05) දිශාව වාමාවර්ත වේ.

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(ii) $M + L - 2P = 2P \rightarrow ①$

$\uparrow 2P \times \frac{\sqrt{3}}{2} + M \times \frac{\sqrt{3}}{2} - L \times \frac{\sqrt{3}}{2} - 6P \times \frac{\sqrt{3}}{2} = 4P \times \frac{\sqrt{3}}{2}$

$2P + M - L - 6P = 4P$

$M - L = 8P \rightarrow ②$ $M = 6P\ N$ (05) $L = -2P\ N$ (05)

පද්ධතියෙහි A හි සූර්ණ,

$2\sqrt{3}\ P \times x = -2P \times \frac{\sqrt{3}}{2} a - 4P \times 2a \times \frac{\sqrt{3}}{2} - 6 \times 2a \times \frac{\sqrt{3}}{2} + 2P \times a \times \frac{\sqrt{3}}{2}$ (15)

$2x = -a - 4a - 6a + a$

$2x = -10a$

$x = -5a$

$\therefore$ B හිම දුර $= 4a$ (05)

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(iii) යුග්මයේ විශාලත්වය

$G = 2\sqrt{3}\,P \times 5a$

$G = 10\sqrt{3}\,Pa$ (10) ; දිශාව ඍජාවර්ත වේ. (05)

A හිදී ක්‍රියාකරන තනි බලය, ඉල් සම්ප්‍රයුක්ත බලයම හා සම දිශාවම වේ (05)

∴ සම්ප්‍රයුක්තයේ විශාලත්වය $= \underline{4P\,N}$ (05)

දිශාව $\overrightarrow{AD}$ ඔස්සේ ක්‍රියා කරයි. (05)

[30]

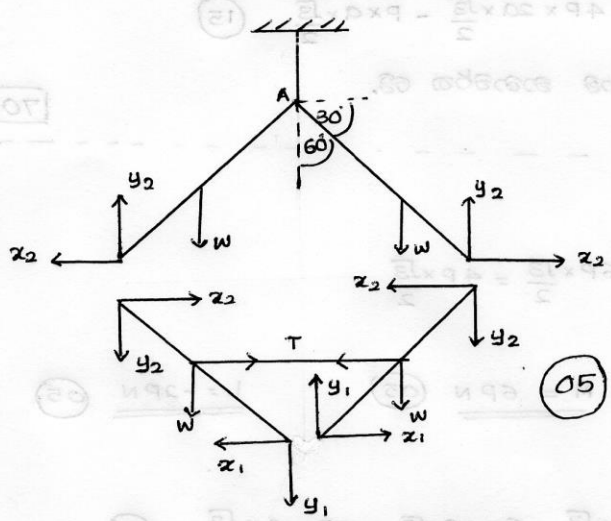
(iv) සම්ප්‍රයුක්තය A හරහා ක්‍රියාකරන පරිදි මෙම තළයට එකතු කළ යුතු යුග්මයේ විශාලත්වය,

$= 2\sqrt{3}P \times 5a$

$= \underline{10\sqrt{3}\,Pa}$ (15) ; දිශාව දක්ෂිණාවර්ත වේ. (05)

[20]

(vi)



(05)

පද්ධතිය AC වටා සමමිතික බැවින්

$y_1 = 0$ (05)

දිග $2a$ ලෙස සලකමු.

BC දණ්ඩට B වටා

$W \times a\cos 30° + x_1 \times 2a\cos 60° + y_1 \times 2a\cos 30° + T \times a\cos 60° = 0$ (10)

$W \times \frac{\sqrt{3}}{2} + 2x_1 \times \frac{1}{2} + 2(0) \times \frac{\sqrt{3}}{2} + \frac{T}{2} = 0$

$\frac{\sqrt{3}W}{2} + x_1 + \frac{T}{2} = 0 \longrightarrow (1)$ (05)

ABC පුර්ධයට A↺

$$w \times a\cos 30^\circ + w \times a\cos 30^\circ + T \times \frac{3a}{2}\cos 60^\circ = x_1 \times 4a\cos 60^\circ \quad (10)$$

$$\frac{\sqrt{3}w}{2} + \frac{\sqrt{3}w}{2} + \frac{3T}{4} = 2x_1 \longrightarrow (2) \quad (05)$$

(1) × 2 + (2)

$$\sqrt{3}w + 2x_1 + T + \frac{\sqrt{3}w}{2} + \frac{\sqrt{3}w}{2} + \frac{3T}{4} = 0 + 2x_1$$

$$2\sqrt{3}w + \frac{7T}{4} = 0$$

$$T = -\frac{8\sqrt{3}w}{7}$$

∴ තෙරපුම $= \frac{8\sqrt{3}}{7}$ (10)

(1) ස්, $x_1 = -\frac{T}{2} - \frac{\sqrt{3}w}{2}$

$$x_1 = \frac{8\sqrt{3}w}{14} - \frac{\sqrt{3}w}{2}$$

$$x_1 = \frac{\sqrt{3}w}{14}$$

∴ C හි ප්‍රතික්‍රියාව $= \frac{\sqrt{3}w}{14}$ තිරස්ව (05)

BC දැන්වේ සමතුලිතතාවය සලකමු.

↑ බල = ↓ බල

$y_1 = w + y_2$

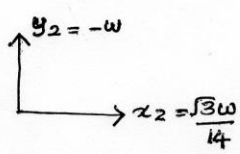
$0 = w + y_2$

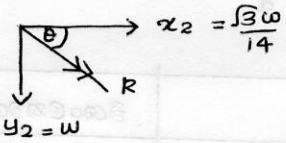
$y_2 = -w$ (05)

→ බල = ← බල

$x_1 = x_2$

$x_2 = \frac{\sqrt{3}w}{14}$ (05)





$$R^2 = w^2 + \frac{3w^2}{196}$$

$$R^2 = \frac{199w^2}{196}$$

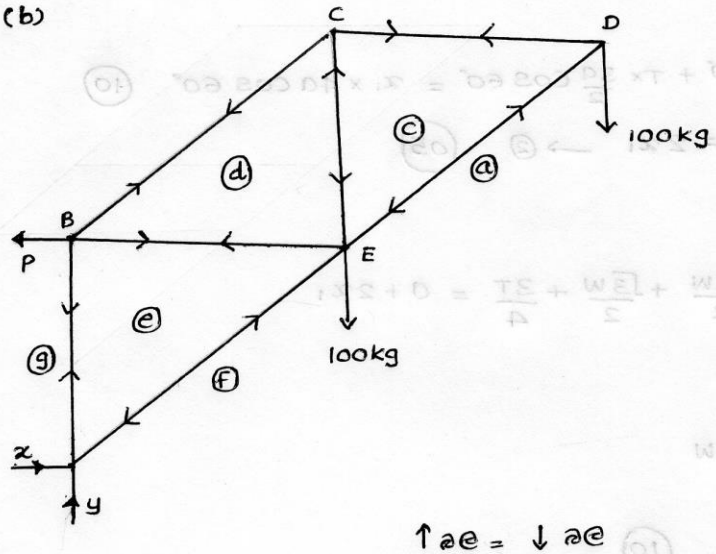
$$R = \frac{\sqrt{199}.w}{14} \quad (05)$$

දිශාව; $\tan\theta = \frac{w}{\sqrt{3}w/14}$

$$\theta = \tan^{-1}\left(\frac{14}{\sqrt{3}}\right) \quad (05)$$

75

(b)



$\uparrow$ බල = $\downarrow$ බල

$y = 200$ kg

(05)

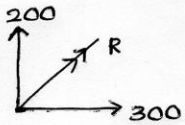
$\rightarrow$ බල = $\leftarrow$ බල

$x = P$

$x = 300$ kg (05)

A $\curvearrowright$ $P = 100 + 200$

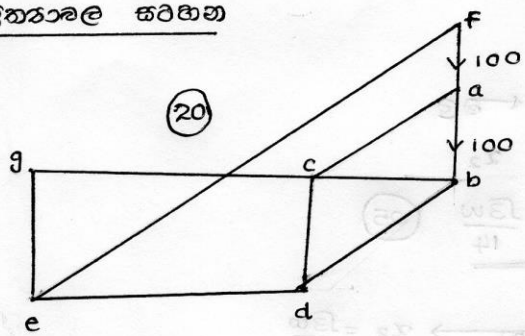
$P = 300$kg (05)



$R = 100 \times \sqrt{9+4}$

$R = 100\sqrt{13}$ kg (05)

ප්‍රතිසාධක සටහන



(20)

දිශ්ඩ	බලය	විශාලත්වය (kg)
AB	ආතති	100 (05)
BC	ආතති	$100\sqrt{2}$ (05)
CD	ආතති	100 (05)
DE	තෙරපුම්	$100\sqrt{2}$ (05)
EA	තෙරපුම්	$300\sqrt{2}$ (05)
EB	ආතති	200 (05)
EC	තෙරපුම්	100 (05)

75