තුන්වන වාර පරීක්ෂණය - 2017

12 ශ්‍රේණිය

සංයුක්ත ගණිතය - I

පිළිතුරු දීමේ පටිපාටිය

A කොටස

(1) $\displaystyle\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{n+2}{2^n}$

$n=1$ විට, L.H.S $= \displaystyle\sum_{r=1}^{1} \frac{r}{2^r}$ R.H.S. $= 2 - \frac{3}{2}$

$= \frac{1}{2^1}$ $= \frac{1}{2}$

$= \frac{1}{2}$

L.H.S = R.H.S ∴ $n=1$ ට ප්‍රතිඵලය සත්‍ය වේ. 05

$n=p$ විට ප්‍රතිඵලය සත්‍ය යැයි ගනිමු. $p \in \mathbb{Z}^+$

$\displaystyle\sum_{r=1}^{p} \frac{r}{2^r} = 2 - \frac{p+2}{2^p}$ 05

දැන් $\displaystyle\sum_{r=1}^{p+1} \frac{r}{2^r} = \sum_{r=1}^{p} \frac{r}{2^r} + \frac{(p+1)}{2^{p+1}}$

$= 2 - \frac{(p+2)}{2^p} + \frac{(p+1)}{2^{p+1}}$ 05

$= 2 - \left(\frac{2(p+2) - (p+1)}{2^{p+1}}\right)$

$= 2 - \frac{(p+3)}{2^{p+1}}$

$= 2 - \frac{(p+1)+2}{2^{p+1}}$ 05

$n=1$ විට ප්‍රතිඵලය සත්‍ය වේ. $n=p$ විට ප්‍රතිඵලය සත්‍ය නම් $n=p+1$ විටද ප්‍රතිඵලය සත්‍ය වේ. ∴ ගණිත අභ්‍යුහන මූලධර්මයට අනුව සියලු ධන නිඛිලමය n සඳහා ප්‍රතිඵලය සත්‍ය වේ. 05

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(2) $f(x) \equiv x^3 + ax^2 + bx + 1$ යයි ගනිමු.

$f(x)$, $x^2 - 3x + 2$ න් බෙදූ විට ලබ්ධිය $Ax + B$ හා ශේෂය $5x+3$ බැවින්,

$$x^3 + ax^2 + bx + 1 \equiv (x^2 - 3x + 2)(Ax + B) + 5x + 3$$

$$x^3 + ax^2 + bx + 1 \equiv (x-2)(x-1)(Ax+B) + 5x + 3 \quad 05$$

$x = 1 \Rightarrow \quad 1 + a + b + 1 = 8$

$a + b = 6 \longrightarrow (1)$

05

$x = 2 \Rightarrow \quad 8 + 4a + 2b + 1 = 13$

$4a + 2b = 4$

$2a + b = 2 \longrightarrow (2)$

05

(2) − (1) $\quad \underline{\underline{a = -4}}$ 05 $\quad \underline{\underline{b = 10}}$ 05

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(3) $$\frac{x^2+5x+2}{x^2(x+1)^2} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} \quad \text{— } 05$$

$$x^2 + 5x + 2 \equiv Ax(x+1)^2 + B(x+1)^2 + Cx^2(x+1) + Dx^2$$

$$x^2 + 5x + 2 \equiv x^3(A+C) + x^2(2A + B + C + D) + x(A + 2B) + B$$

සංගුණක සමාන කිරීම,

x^0 ; $\quad 2 = B$ 05

x^1 ; $\quad 5 = A + 2B$

$A = 1$ 05

x^3 ; $\quad 0 = A + C$

$C = -1$ 05

x^2 ; $\quad 1 = 2A + B + C + D$

$D = -2$ 05

$$\frac{x^2+5x+2}{x^2(x+1)^2} \equiv \frac{1}{x} + \frac{2}{x^2} - \frac{1}{(x+1)} - \frac{2}{(x+1)^2}$$

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(4) $2x^2 + px + q = 0$ හි මූල α හා β ද,

$x^2 + qx + p = 0$ හි මූල α හා γ ලෙසද ගනිමු.

එවිට පොදු මූලය α වේ.

$2\alpha^2 + p\alpha + q = 0 \longrightarrow (1) \qquad \alpha^2 + q\alpha + p = 0 \longrightarrow (2)$

(1) − (2)×2

$(p - 2q)\alpha + q - 2p = 0$

$\alpha = \dfrac{2p - q}{p - 2q}$; $\quad p \neq 2q$ 05

$\alpha\beta = \dfrac{q}{2}$ 05

$\beta = \dfrac{q}{2\alpha}$

$\beta = \dfrac{q(p-2q)}{2(2p-q)}$

05

$\alpha\gamma = p$ 05

$\gamma = \dfrac{p}{\alpha}$

$\gamma = \dfrac{p(p-2q)}{(2p-q)}$

05

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(5) $\log_{25} x^2 + (\log_5 x)^2 = 2$

$2\log_{25} x + (\log_5 25 \cdot \log_{25} x)^2 = 2$ (05)

$2\log_{25} x + (2\log_{25} x)^2 = 2$ (05)

$2y + 4y^2 = 2$, $y = \log_{25} x$

$2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$y = \frac{1}{2}$ හෝ $y = -1$ (05)

$y = \frac{1}{2} \Rightarrow \log_{25} x = \frac{1}{2}$, $x = 25^{1/2} = 5$ (05)

$y = -1 \Rightarrow \log_{25} x = -1$, $x = 25^{-1} = \frac{1}{25}$ (05)

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(6) $\frac{3}{2-x} > 4$

$x = 2$ විට, අසමානතාව අර්ථ නොදැක්වේ. (05)

$x \neq 2$ විට, $\frac{3}{2-x} - 4 > 0$

$\frac{3 - 4(2-x)}{(2-x)} > 0$

$\frac{4x-5}{2-x} > 0$ (05)

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5/4 2

x	$\frac{4x-5}{2-x}$
$x < \frac{5}{4}$	$(-)$
$x = \frac{5}{4}$	$= 0$
$\frac{5}{4} < x < 2$	$(+)$
$x > 2$	$(-)$

(10)

$\frac{3}{2-x} > 4 \Rightarrow \frac{4x-5}{2-x} > 0$

$\Rightarrow \left\{ x \mid \frac{5}{2} < x < 2 \right\}$ (05)

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(07) $$l = \lim_{x\to 0} \frac{\sqrt{1+4x^2} - \sqrt{1+x^2}}{\cos 5x - \cos 3x}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+4x^2} - \sqrt{1+x^2}}{(-2)\sin 4x \sin x} \times \frac{\sqrt{1+4x^2} + \sqrt{1+x^2}}{\sqrt{1+4x^2} + \sqrt{1+x^2}}$$ (05) (05)

$$= \lim_{x\to 0} \frac{1+4x^2 - (1+x^2)}{(-2)\sin 4x \sin x\left(\sqrt{1+4x^2}+\sqrt{1+x^2}\right)}$$

$$= \lim_{x\to 0} \frac{1+4x^2-1-x^2}{(-2)\sin 4x \cdot \sin x \cdot \left(\sqrt{1+4x^2}+\sqrt{1+x^2}\right)}$$

$$= \lim_{x\to 0} \frac{3x^2}{(-2)\sin 4x \sin x\left(\sqrt{1+4x^2}+\sqrt{1+x^2}\right)}$$

$$= \lim_{x\to 0} \frac{-3}{2\times 4} \frac{1}{\frac{\sin 4x}{4x}\cdot\frac{\sin x}{x}\left(\sqrt{1+4x^2}+\sqrt{1+x^2}\right)}$$ (05)

$$= \lim_{x\to 0} \frac{-3}{8\left(\sqrt{1+4x^2}+\sqrt{1+x^2}\right)\times \lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)\lim_{x\to 0}\left(\frac{\sin x}{x}\right)}$$ (05)

$$= \frac{-3}{8(1+1)\times 1\times 1}$$

$$= \underline{\underline{\frac{-3}{16}}}$$ (05)

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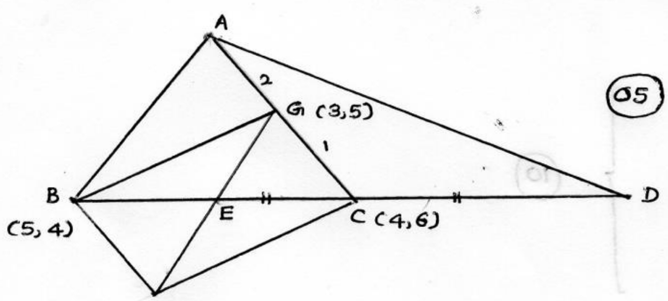
(08)



G, AC මධ්‍යස්ථය මත AG : GC = 2 : 1 වන සේ පිහිටයි. (05)

$$\therefore G \equiv \left\{\frac{8+1}{3}, \frac{12+3}{3}\right\} \equiv (3, 5)$$ (05)

E යනු BC හි මධ්‍ය ලක්ෂ්‍යයයි. (05)

$$E \equiv \left\{\frac{5+4}{2}, \frac{4+6}{2}\right\} \equiv \left(\frac{9}{2}, 5\right)$$ (05)

$\boxed{25}$

(09) $\cos 2\theta = \frac{-7}{25}$

$$2\cos^2\theta - 1 = \frac{-7}{25} \quad (05)$$

$$2\cos^2\theta = \frac{-7}{25} + 1$$

$$\cos^2\theta = \frac{9}{25}$$

$\cos\theta = \frac{3}{5}$ ($\because 0 < \theta < \pi/2$ විට $\cos\theta > 0$) (05)

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \frac{9}{25}$$

$$= \frac{16}{25}$$

$\sin\theta = \frac{4}{5}$ ($\because 0 < \theta < \pi/2$ විට $\sin\theta > 0$) (05)

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3} \quad (05)$$

$$\sin\theta + \cos\theta + \tan\theta = \frac{3}{5} + \frac{4}{5} + \frac{4}{3} \Rightarrow \frac{9+12+20}{15} = \underline{\underline{\frac{41}{15}}} \quad (05)$$

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(10) සුපුරුදු ලෙස අංකනය කළ ඕනෑම ත්‍රිකෝණයක් සඳහා

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (05)$$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$ නම්,

$$a = k\sin A \ , \ b = k\sin B \ , \ c = k\sin C$$

$$a = b + \lambda c \Rightarrow a - b = \lambda c$$

$$k\sin A - k\sin B = \lambda . k . \sin C \quad (05)$$

$$\sin A = \sin(\pi - (B+C))$$

$$\sin A = \sin(B+C) \quad (05)$$

$$\sin(B+C) - \sin B = \lambda . 2\sin\frac{C}{2}\cos\frac{C}{2} \quad (05)$$

$$2\cos\left(B + \frac{C}{2}\right)\sin\frac{C}{2} = 2\lambda . \sin\frac{C}{2} . \cos\frac{C}{2} \quad (05)$$

$$\underline{\underline{\cos\left(B + \frac{C}{2}\right) = \lambda . \cos\frac{C}{2}}}$$

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B කොටස

(11) (a) $f(x) \equiv ax^2+bx+c$, $a, b, c \in \mathbb{R}$, $a \neq 0$.

$$f(x) \equiv a\left(x+\frac{b}{a}+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right)$$

$$\equiv a\left\{ \left(x+\frac{b}{2a}\right)^2-\frac{(b^2-4ac)}{4a^2}\right\}$$

$$\equiv a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a}\right) \quad (20)$$

$$a>0 \Rightarrow a\left(x+\frac{b}{2a}\right)^2 \geq 0$$

$$a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a}\right) \geqslant -\left(\frac{b^2-4ac}{4a}\right)$$

$$f(x) \geqslant -\frac{(b^2-4ac)}{4a} \quad (20)$$

$a>0$ හා $b^2-4ac<0 \Rightarrow -\frac{(b^2-4ac)}{4a}>0$ (05)

$$f(x) \geqslant -\frac{(b^2-4ac)}{4a}>0 \quad \therefore f(x)>0 \quad (05)$$

$$(f(x))_{min} = -\left(\frac{b^2-4ac}{4a}\right) \quad (05)$$

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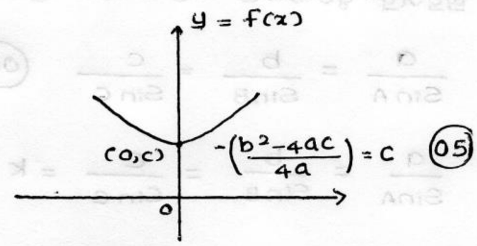
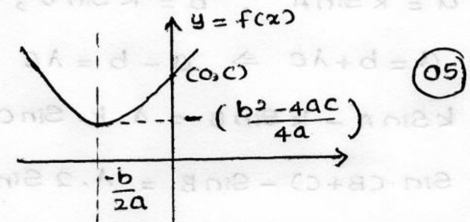
$f(x)$ අඩුතම විට $\left(x+\frac{b}{2a}\right)^2=0$

$$x=-\frac{b}{2a} \quad (05)$$

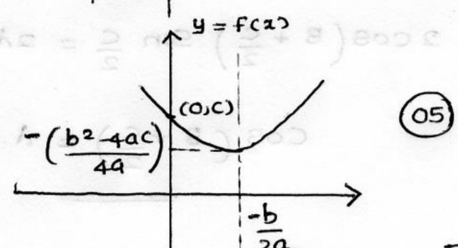
$b=0 \Rightarrow -\frac{b}{2a}=0$



$b>0 \Rightarrow -\frac{b}{2a}<0$



$b<0 \Rightarrow -\frac{b}{2a}>0$



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(b) $f(x) \equiv x^2 - 8x + \lambda^2 - 6\lambda \qquad \lambda \in \mathbb{R}$

මෙහි විවේචකය ; $\Delta x = 64 - 4(\lambda^2 - 6\lambda)$ (10)

$= -4(\lambda^2 - 6\lambda - 16)$

$= -4(\lambda - 8)(\lambda + 2)$ (10)

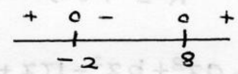
$f(x) = 0$ ට තාත්වික ප්‍රභින්න මූල 2ක් ඇත්නම්,

$\Delta x > 0$ (05)

$\Rightarrow -4(\lambda - 8)(\lambda + 2) > 0$

$\Rightarrow (\lambda - 8)(\lambda + 2) < 0$

$\Rightarrow \underline{-2 < \lambda < 8}$ (05)



$f(x) = 0$ හි මූල α හා β නම්,

$\alpha + \beta = -(-8) = 8 > 0$ (05)

$\alpha + \beta > 0$ බැවින් මූල දෙකම සෘණ විය නොහැක (05)

$f(x) = 0$ හි මූල දෙකම ධන නම්,

$\alpha + \beta > 0$ හා $\alpha\beta > 0$ (05), $\therefore \alpha + \beta > 0$ බැවින් $\alpha\beta > 0$ ද විය යුතුය. (05)

$\lambda^2 - 6\lambda > 0$

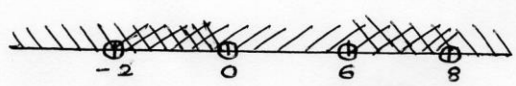
$\lambda(\lambda - 6) > 0$ (05)

$\lambda < 0$ හෝ $\lambda > 6$ විය යුතුය. (05)

මූල තාත්විකද විය යුතු බැවින්,

$-2 < \lambda < 8$ ද විය යුතුය

(05)



(05) (05)

$-2 < \lambda < 0$ හෝ $6 < \lambda < 8$ විය යුතුය.

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(12) ශේෂ ප්‍රමේයය :

$f(x)$; x හි බහුපද ශ්‍රිතයකි. $x \in \mathbb{R}$ විට, $(x-\alpha)$ න්, $f(x)$ බෙදූ විට ශේෂය $f(\alpha)$ වේ. (05)

සාධනය :

$(x-\alpha)$ න්, $f(x)$ බෙදූ විට ලබ්ධිය $g(x)$ ද ශේෂය R ද නම්,

$$f(x) \equiv (x-\alpha)g(x) + R \quad (05)$$

$$x=\alpha \Rightarrow f(\alpha) = 0 \cdot g(\alpha) + R$$

$$R = f(\alpha) \quad (05)$$

$$f(x) \equiv x^4 + ax^3 + bx^2 - 17x + 6$$

$(x-1)^2$, $f(x)$ හි සාධකයක් නම්,

$$x^4 + ax^3 + bx^2 - 17x + 6 \equiv (x-1)^2(px^2+qx+r) \quad (05)$$

$$x^4 + ax^3 + bx^2 - 17x + 6 \equiv (x^2-2x+1)(px^2+qx+r)$$

සංගුණක සමාන කිරීමෙන්,

x^4 ; $1 = p \rightarrow ①$ (05)

x^3 ; $a = q - 2p$

$a = q - 2 \rightarrow ②$ (05)

x^2 ; $b = r - 2q + p$

$b = r - 2q + 1 \rightarrow ③$ (05)

x^1 ; $-17 = -2r + q \rightarrow ④$ (05)

x^0 ; $6 = r \rightarrow ⑤$ (05)

$q = -5$

$a = -7$

$b = 6 + 10 + 1 = 17$

$\therefore \underline{\underline{a = -7}}$ (05) , $\underline{\underline{b = 17}}$ (05)

$$f(x) \equiv (x-1)^2(x^2-5x+6)$$

$$\equiv (x-1)^2(x-2)(x-3) \quad (05)$$

$\therefore$ බහුපදයේ ඉතිරි සාධක වනුයේ $(x-2)(x-3)$ ය. (05)

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(b) $$\frac{1}{(x-\lambda)(x-\mu)} = \frac{k}{(x-\lambda)} + \frac{l}{(x-\mu)}$$

$$1 \equiv k(x-\mu) + l(x-\lambda)$$

$$1 \equiv (k+l)x - (k\mu + l\lambda)$$

සංගුණක සැසඳීම.

$$x^1 : \quad 0 = k + l \longrightarrow (1)$$

$$x^0 : \quad 1 = -(k\mu + l\lambda)$$

$$-1 = k\mu + l\lambda \longrightarrow (2)$$

$(1) \times \lambda - (2)$

$$(\lambda - \mu)k = 1$$

$$k = \frac{1}{(\lambda-\mu)} \quad 05 \qquad\qquad l = \frac{-1}{(\lambda-\mu)} \quad 05$$

$$\frac{1}{(x-\lambda)(x-\mu)} \equiv \frac{1}{(\lambda-\mu)(x-\lambda)} - \frac{1}{(\lambda-\mu)(x-\mu)}$$

$$\frac{1}{(x-\lambda)^2(x-\mu)^2} \equiv \left\{\frac{1}{(\lambda-\mu)(x-\lambda)} - \frac{1}{(\lambda-\mu)(x-\mu)}\right\}^2$$

$$\equiv \frac{1}{(\lambda-\mu)^2(x-\lambda)^2} + \frac{1}{(\lambda-\mu)^2(x-\mu)^2} - \frac{2}{(\lambda-\mu)^2(x-\lambda)(x-\mu)} \quad 05$$

$$\equiv \frac{1}{(\lambda-\mu)^2(x-\lambda)^2} + \frac{1}{(\lambda-\mu)^2(x-\mu)^2} - \frac{2}{(\lambda-\mu)^2}\left\{\frac{1}{(\lambda-\mu)(x-\lambda)} - \frac{1}{(\lambda-\mu)(x-\mu)}\right\}$$

05

$$\equiv \frac{1}{(\lambda-\mu)^2(x-\lambda)^2} + \frac{1}{(\lambda-\mu)^2(x-\mu)^2} - \frac{2}{(\lambda-\mu)^3(x-\lambda)} + \frac{2}{(\lambda-\mu)^3(x-\mu)} \quad 05$$

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(c) $$x^2 + 6x + 11 \equiv x^2 + 6x + 9 + 2$$

$$\equiv (x+3)^2 + 2 \quad 05$$

$\forall x \in \mathbb{R}$ සඳහා $(x+3)^2 \geq 0$

$$(x+3)^2 + 2 \geq 2$$

$$x^2 + 6x + 9 + 2 \geq 2 > 0$$

$$x^2 + 6x + 11 > 0 \quad 05$$

$$\frac{(x-2)(x^2+6x+11)}{(x^2-6x+5)} \geq 0 \Rightarrow \frac{(x-2)}{(x-5)(x-1)} \geq 0 \quad (05)$$

$-$	$*$	$+$	0	$-$	$*$	$+$
	1		2		5	

$x = 5$ හෝ $x = 1$ විට අසමානතාව අර්ථ නොදැක්වේ.

(05) (05)

x හි අගය	$\frac{(x-2)}{(x-5)(x-1)}$ හි ලකුණ	
$x < 1$	$(-)$	(05)
$1 < x < 2$	$(+)$	(05)
$x = 2$	$= 0$	(05)
$2 < x < 5$	$(-)$	(05)
$x > 5$	$(+)$	(05)

විසඳුම :- $1 < x \leq 2$ හෝ $x > 5$

(05) (05)

$\boxed{60}$

(13) (a) $f(n) = 7^n - 1 \ ; \ n \in \mathbb{Z}^+$

$n = 1$ විට , $f(n) = 7^1 - 1 = 6 = 6 \times 1$

$\therefore$ $n = 1$ විට $f(n)$, 6 න් බෙදේ (10)

$n = p$ විට, $f(n)$, 6 න් බෙදේ යැයි ගනිමු. $p \in \mathbb{Z}^+$

$f(p) = 6k \ ; \ k \in \mathbb{Z}^+ \longrightarrow ①$ (10)

$f(p) = 7^p - 1 \longrightarrow ②$ (10)

$f(p+1) = 7^{p+1} - 1 \longrightarrow ③$ (10)

③ $- 7 \times$ ② න්

$f(p+1) - 7f(p) = 7^{p+1} - 1 - 7^{p+1} + 7$

$f(p+1) - 7 \times 6k = 6$

$f(p+1) = 6(1 + 7k) = 6k'$ (10)

$f(p)$, 6න් බෙදේනම්, $f(p+1)$ ත් 6න් බෙදේ.

$\therefore$ ගණිත අභ්‍යුහන මූලධර්මයට අනුව සියලු $n \in \mathbb{Z}^+$ සඳහා $f(n)$, 6 න් බෙදේ. (10)

$\boxed{60}$

(b) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ (10)

$\therefore e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$ (05)

$3(r+1)(r+2) + 5(r+2) + 1 \equiv 3(r^2+3r+2) + 5r + 10 + 1$

$\equiv 3r^2 + 14r + 17$ (05)

$3r^2 + 14r + 17 \equiv A(r^2+3r+2) + B(r+2) + C$ (05)

r^2 හි සංගුණක : $3 = A$ (05)

r හි 〃 : $3A + B = 14 \Rightarrow B = 5$ (05)

r^0 හි 〃 : $2A + 2B + C = 17 \Rightarrow C = 1$ (05)

$\therefore \frac{3r^2+14r+17}{(r+2)!} = \frac{3}{r!} + \frac{5}{(r+1)!} + \frac{1}{(r+2)!}$ (05)

$U_r = \frac{3}{r!} + \frac{5}{(r+1)!} + \frac{1}{(r+2)!}$ ලෙස ගනිමු.

$U_1 = \frac{3}{1} + \frac{5}{2!} + \frac{1}{3!}$ (05)

$U_2 = \frac{3}{2!} + \frac{5}{3!} + \frac{1}{4!}$ (05)

$U_3 = \frac{3}{3!} + \frac{5}{4!} + \frac{1}{5!}$ (05)

$U_4 = \frac{3}{4!} + \frac{5}{5!} + \frac{1}{6!}$ (05)

$\vdots$ +

$\sum_{r=1}^{n} U_r = 3\left(1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots\right) + 5\left(\frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots\right) + 1\left(\frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots\right)$ (05)

$= 3(e-1) + 5(e-2) + 1\left(e - \frac{5}{2}\right)$ (05) (05) (05)

$= 9e - \frac{31}{2}$ (05)

90

(14) (a) $\cos 3x = 4\cos^3 x - 3\cos x$ 05

$$\cos 6x \equiv 2\cos^2 3x - 1 \quad 05$$

$$\equiv 2\left\{4\cos^3 x - 3\cos x\right\}^2 - 1 \quad 05$$

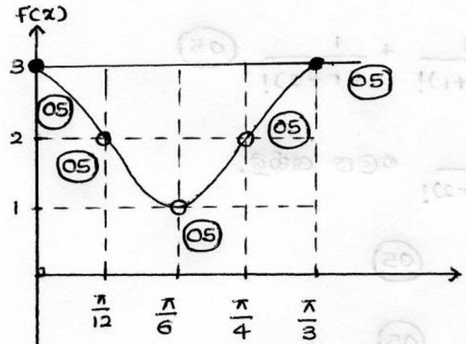
$$\equiv 2\left\{16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right\} - 1 \quad 05$$

$$\equiv 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

(i) $f(x) \equiv 32\cos^6 x - 48\cos^4 x + 18\cos^2 x + 1$

$$\equiv 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1 + 2$$

$$\equiv \cos 6x + 2 \quad 05$$



හැඩය හා සුමට බවට 05

අක්ෂ නිවැරදි ක්‍රමාංකනයට 05

60

(ii) $\left(32\cos^6 x - 48\cos^4 x + 18\cos^2 x\right) + 2\sin 3x \cos 3x = 0$

$$\cos 6x + 1 + \sin 6x = 0 \quad 05$$

$$\sqrt{2}\left(\cos 6x \cdot \frac{1}{\sqrt{2}} + \sin 6x \cdot \frac{1}{\sqrt{2}}\right) = -1 \quad 05$$

$$\cos 6x . \cos\frac{\pi}{4} + \sin 6x . \sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$$

$$\cos\left(6x - \pi/4\right) = \cos\frac{3\pi}{4} \quad 05$$

$$6x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4} \;;\; n \in \mathbb{Z}^+ \quad 05$$

(+) $$6x - \frac{\pi}{4} = 2n\pi + \frac{3\pi}{4} \quad 05$$

$$6x = 2n\pi + \frac{3\pi}{4} + \frac{\pi}{4}$$

$$6x = 2n\pi + \pi$$

$$x = \frac{\pi}{6}(2n+1) \;;\; n \in \mathbb{Z} \quad 05$$

(ii) $6x - \frac{\pi}{4} = 2n\pi - \frac{3\pi}{4}$ (05)

$6x = 2n\pi - \frac{\pi}{2}$

$6x = \pi\left(2n - \frac{1}{2}\right)$

$x = \frac{\pi}{12}(4n-1) \; ; \; n \in Z^{+}$ (05) $\boxed{40}$

(b) $\tan^{-1}\left(\frac{1}{3}\right) = \alpha \Rightarrow \tan\alpha = \frac{1}{3} \rightarrow ①$ (05) $\quad 0 < \alpha < \frac{\pi}{6} \rightarrow ②$ (05)

$\tan^{-1}\left(\frac{1}{4}\right) = \beta \Rightarrow \tan\beta = \frac{1}{4} \rightarrow ③$ (05) $\quad 0 < \beta < \frac{\pi}{6} \rightarrow ④$ (05)

$\tan^{-1}\left(\frac{2}{9}\right) = \gamma \Rightarrow \tan\gamma = \frac{2}{9} \rightarrow ⑤$ (05) $\quad 0 < \gamma < \frac{\pi}{6} \rightarrow ⑥$ (05)

② + ④ + ⑥ න්

$0 < \alpha + \beta + \gamma < \frac{\pi}{2} \rightarrow Ⓐ$ (05)

$\alpha + \beta + \gamma = \theta$

$\alpha + \beta = \theta - \gamma$

$\tan(\alpha+\beta) = \tan(\theta - \gamma)$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\tan\theta - \tan\gamma}{1 + \tan\theta\tan\gamma}$$

$$\frac{\frac{1}{3} + \frac{1}{4}}{1 - \frac{1}{3} \times \frac{1}{4}} = \frac{\tan\theta - 2/9}{1 + \tan\theta \times 2/9}$$

$$\frac{7}{11} = \frac{9\tan\theta - 2}{9 + 2\tan\theta}$$

$63 + 14\tan\theta = 99\tan\theta - 22$

$85 = 85\tan\theta$

$\tan\theta = 1$ (10)

$\therefore \tan(\alpha+\beta+\gamma) = 1 \rightarrow Ⓑ$

Ⓐ හා Ⓑ න්, $\alpha + \beta + \gamma = \frac{\pi}{4} \rightarrow$ (05)

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{\pi}{4}$ $\boxed{50}$

(15) (a) A, B, C Δ ක $\angle$ නම්,

$$A+B+C=\pi \quad (05)$$

$$A+(B+\theta)+(C-\theta)=\pi \quad (05)$$

$$A+(B+\theta)=\pi-(C-\theta) \quad (05)$$

$$\tan\{A+(B+\theta)\}=\tan(\pi-(C-\theta)) \quad (05)$$

$$\frac{\tan A+\tan(B+\theta)}{1-\tan A\tan(B+\theta)}=-\tan(C-\theta) \quad (05)$$

$$\tan A+\tan(B+\theta)=-\tan(C-\theta)+\tan A\tan B\tan(C-\theta) \quad (05)$$

$$\tan A+\tan(B+\theta)+\tan(C-\theta)=\tan A\tan B\tan(C-\theta)\rightarrow \circledast$$

$$\frac{\pi}{3}+\frac{5\pi}{12}+\frac{\pi}{4}=\pi \quad (05)$$

$\circledast$ හි $A=\frac{\pi}{3}$, $B=\frac{5\pi}{12}$, $C=\frac{\pi}{4}$ හා $\theta=0$ යොදමු. (05)

$$\tan\frac{\pi}{3}+\tan\frac{5\pi}{12}+\tan\frac{\pi}{4}=\tan\frac{\pi}{3}\tan\frac{5\pi}{12}\tan\frac{\pi}{4}$$

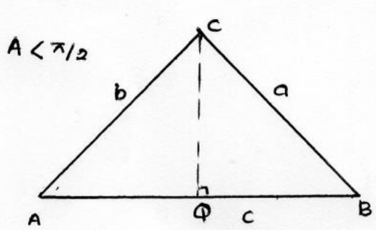
$\boxed{40}$

(b) සුපුරුදු ලෙස අංකනය කළ ඕනෑම ABC Δ ක් සඳහා

$$\left.\begin{aligned} a^2&=b^2+c^2-2bc\cos A\\ b^2&=a^2+c^2-2ac\cos B\\ c^2&=a^2+b^2-2ab\cos C\end{aligned}\right\}\ (10) \qquad \text{එකක් නම් } (05)$$

$\boxed{10}$

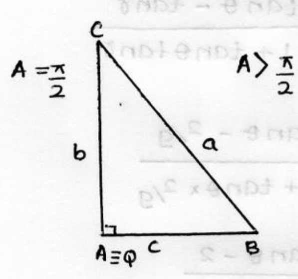
සාධනය :-



$C\Phi=b\sin A$

$\Phi B=c-b\cos A$

(05)



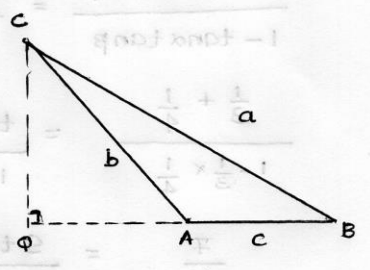
$C\Phi=b=b\sin\frac{\pi}{2}$

$C\Phi=b\sin A$

$\Phi B=c=c-b\cos\frac{\pi}{2}$

$\Phi B=c-b\cos A$

(05)



$C\Phi=b\sin(\pi-A)$

$=b\sin A$

$B\Phi=c+b\cos(\pi-A)$

$=c-b\cos A$

(05)

අවස්ථා ප්‍රමේයය CΦB Δ න්.

$$BC^2 = C\Phi^2 + \Phi B^2 \quad (05)$$

$$a^2 = (b\sin A)^2 + (c - b\cos A)^2$$

$$= b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A \quad (05)$$

$$\underline{\underline{a^2 = b^2 + c^2 - 2bc\cos A}} \quad (05)$$

$\boxed{30}$

A, B, C සමාන්තර ශ්‍රේණියක නම්,

$$C - B = B - A$$

$$A + C = 2B \quad (05)$$

තවද, $A + B + C = \pi$

$$3B = \pi$$

$$B = \frac{\pi}{3} \quad (05)$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad (05)$$

$$\frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$$

(05)

$$ac = a^2 + c^2 - b^2$$

$$b^2 = a^2 + c^2 - ac$$

$$b = \sqrt{a^2 + c^2 - ac} \;;\; (b > 0) \quad (05)$$

(ii) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$ නම් (10)

$$\frac{a+c}{b} = \frac{k\sin A + k\sin C}{k\sin B} \quad (15) \Rightarrow (05) \times 3$$

$$\frac{a+c}{b} = \frac{\sin A + \sin C}{\sin B} \quad (05)$$

$$\frac{a+c}{b} = \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}{\sin B.} \quad (05) \qquad A+B+C = \pi$$

$$\frac{a+c}{\sqrt{a^2+c^2-ac}} = \frac{2\sin B.\cos\left(\frac{A-C}{2}\right)}{\sin B} \quad (05)$$

$$\underline{\underline{\frac{a+c}{\sqrt{a^2+c^2-ac}} = 2\cos\left(\frac{A-C}{2}\right)}} \quad (05)$$

$\boxed{70}$

(16) (a) $y = f(x) = \sin x$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \quad 05$$

$$= \lim_{\delta x \to 0} \frac{\sin(x+\delta x) - \sin x}{\delta x} \quad 05$$

$$= \lim_{\delta x \to 0} \frac{2\cos\left(x+\frac{\delta x}{2}\right) \sin\frac{\delta x}{2}}{\delta x} \quad 05$$

$$= \lim_{\delta x \to 0} \underbrace{\cos\left(x+\frac{\delta x}{2}\right)}_{05} \times \lim_{\delta x \to 0} \frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}} \quad 05$$

$$= \cos x \times 1$$

$$\frac{dy}{dx} = \cos x \quad 05$$

$\boxed{30}$

$y = \sin^{-1} x \Rightarrow x = \sin y$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ 05

$$\frac{dx}{dy} = \cos y \longrightarrow (*) \quad 05$$

$$\cos^2 y = 1 - \sin^2 y = 1 - x^2$$

$$\cos y = \pm\sqrt{1 - x^2} \quad 05$$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ බැවින් $\cos y \geq 0$ 05

$\therefore \cos y = \sqrt{1-x^2}$ 05

$\therefore$ (*) න්, $\frac{dx}{dy} = \sqrt{1-x^2}$ 05

$$\frac{dy}{dx} = \frac{1}{dx/dy} \quad 05$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad 05$$

$\boxed{40}$

$$y = x\sin^{-1}x$$

$$\frac{dy}{dx} = x.\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \quad (10)$$

$$\sqrt{1-x^2}\ \frac{dy}{dx} = x + \sqrt{1-x^2}.\sin^{-1}x.$$

$$\sqrt{1-x^2}\ \frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{1}{2}.\frac{1}{\sqrt{1-x^2}}.(-2x) = 1 + \sqrt{1-x^2}\times\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \times \frac{(-2x)}{2\sqrt{1-x^2}} \quad (10)$$

$$(1-x^2)\frac{d^2y}{dx^2} - x.\frac{dy}{dx} = 2\sqrt{1-x^2} - x.\sin^{-1}x \quad (5)$$

$$(1-x^2)\frac{d^2y}{dx^2} - x.\frac{dy}{dx} = 2\sqrt{1-x^2} - y \quad (5)$$

$$(1-x^2)\frac{d^2y}{dx^2} - x.\frac{dy}{dx} + y = 2\sqrt{1-x^2}$$

$\boxed{30}$

(b) $y = e^t\cos t$ $\qquad$ $x = e^t\sin t$

$$\frac{dy}{dt} = e^t(-\sin t) + \cos t.e^t \quad (05)$$

$$= -x + y$$

$$= y - x \quad (05)$$

$$\frac{dx}{dt} = e^t(\cos t) + \sin t.e^t \quad (05)$$

$$= y + x \quad (05)$$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

$$\frac{dy}{dx} = (y-x)\times\frac{1}{(x+y)}$$

$$(x+y).\frac{dy}{dx} = y - x \longrightarrow (1) \quad (05)$$

(1) $\times (x+y)$

$$(x+y)^2.\frac{dy}{dx} = y^2 - x^2$$

$$(x+y)^2.\frac{d^2y}{dx^2} + \frac{dy}{dx}.2(x+y)\left\{1 + \frac{dy}{dx}\right\} = 2y.\frac{dy}{dx} - 2x$$

$$(x+y)^2.\frac{d^2y}{dx^2} + (y-x)\left(2 + 2.\frac{dy}{dx}\right) = 2y.\frac{dy}{dx} - 2x$$

$$(x+y)^2\frac{d^2y}{dx^2}+2y+2y.\frac{dy}{dx}-2x-2x\cdot\frac{dy}{dx}=2y.\frac{dy}{dx}-2x$$

$$(x+y)^2\frac{d^2y}{dx^2}=2\left(x\cdot\frac{dy}{dx}-y\right) \quad (15)$$

$\boxed{40}$

(c) $2x^2-xy-y^2+3y-4=0$

$$4x-\left\{x\cdot\frac{dy}{dx}+y\right\}-2y\cdot\frac{dy}{dx}+3\cdot\frac{dy}{dx}=0$$

$$4x-y=(x+2y-3)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{4x-y}{x+2y-3} \quad (10)$$

$\boxed{10}$

* $\left(\frac{dy}{dx}\right)_{x=0}$ සෙවීම සිදුකළ නොහැක. ඉහත පියවර ඇත්නම් ලකුණු ලබාදෙන්න.

$2x^2-xy+y^2+3y-4=0$ නම්, $\left(\frac{dy}{dx}\right)_{x=0}$ සෙවිය හැක

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(17) $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$

$$x^2+x+1=x^2+x+\frac{1}{4}-\frac{1}{4}+1$$

$$=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\geq\frac{3}{4} \quad (10)$$

$$x^2+x+1\neq 0$$

$\therefore\ \forall x\in\mathbb{R}$ සඳහා $f(x)$ අර්ථ දැක්වේ.

$$\lim_{x\to\pm\infty}f(x)=\lim_{x\to\pm\infty}\left(\frac{x^2-x+1}{x^2+x+1}\right)$$

$$=\lim_{x\to\pm\infty}\left(\frac{1-1/x+1/x^2}{1+1/x+1/x^2}\right)$$

$$=1 \quad (10)$$

$y=1$ තිරස් ස්පර්ශෝන්මුඛයකි. (05)

$x = 0$ විට $f(x) = 1$ (05)

$f(x) = 0$ විට $x^2 - x + 1 = 0$, $\Delta x < 0$ බැවින් මූල නැත. (05)

$(0, 1)$ හරහා යයි.

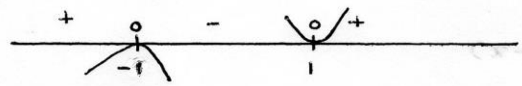
$$f'(x) = \frac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{2x^3 + 2x^2 + 2x - x^2 - x - 1 - 2x^3 + 2x^2 - 2x - x^2 + x - 1}{(x^2+x+1)^2}$$

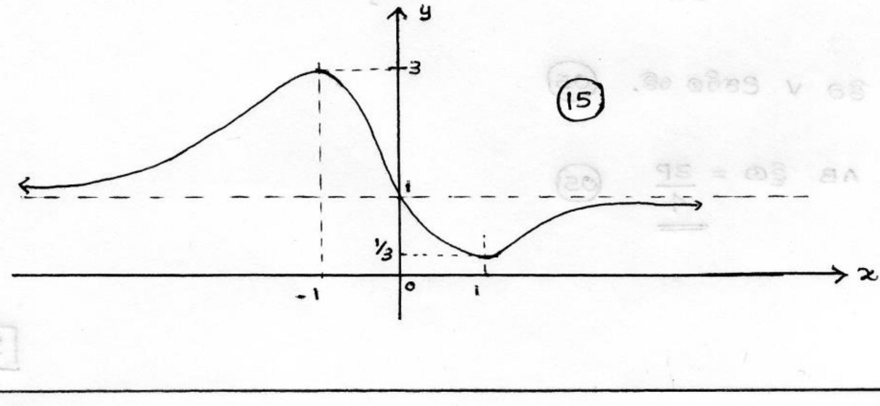
$$= \frac{2x^2 - 2}{(x^2+x+1)^2}$$

$$= \frac{2(x-1)(x+1)}{(x^2+x+1)^2} \quad (15)$$

$x = \pm 1$ විට, $f'(x) = 0$



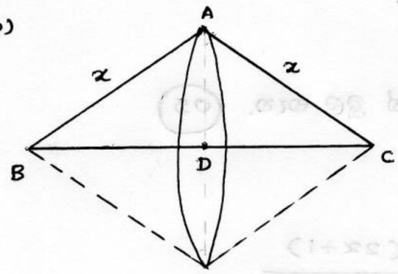
x	$f'(x)$	$f(x)$	ස්.ල. ස්වභාවය	ස්.ල. ඛණ්ඩාංක
$x < 1$	$(+)$	නිතිමතින් වැඩිවේ (05)		
$x = -1$	$= 0$		සාපේක්ෂ උපරිමයකි. (05)	$(-1, 3)$ (05)
$-1 < x < 1$	$(-)$	නිතිමතින් අඩුවේ (05)		
$x = 1$	$= 0$		සාපේක්ෂ අවමයකි. (05)	$(1, 1/3)$ (05)
$x > 1$	$(+)$	නිතිමතින් වැඩිවේ. (05)		



(15)

100

(b)



$AB = x \Rightarrow x + x + BC = 2P$

$2x + 2BD = 2P$

$BD = P - x$ 05

$AD^2 = x^2 - (P-x)^2$ 05

පරිමාව V නම්, $V = \frac{1}{3}\pi r^2 h \times 2$

$= \frac{1}{3}\pi \left\{ x^2 - (P-x)^2 \right\} (P-x) \times 2$ 05

$= \frac{2}{3}\pi (2Px - P^2)(P-x)$ 05

ඉහත V අර්ථ දැක්වෙන්නේ

$2Px - P^2 > 0$ හා $P - x > 0$ විටය.

$x > \frac{P}{2}$ හා $x < P$

එනම්, $\frac{P}{2} < x < P$ විටයි. 05

$V = \frac{2}{3}\pi P (2x - P)(P - x)$

$= \frac{2}{3}\pi P (-2x^2 + 3Px - P^2)$

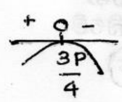
$V = -\frac{2}{3}\pi P (2x^2 - 3Px + P^2)$ 05

$\frac{dV}{dx} = \frac{-2\pi P}{3}(4x - 3P)$ 05

$\frac{dV}{dx} = 0 \Rightarrow x = \frac{3P}{4}$ පිළිගත හැක.

$\frac{P}{2} < x < \frac{3P}{4} \Rightarrow \frac{dV}{dx} > 0$

$\frac{3P}{4} < x < P \Rightarrow \frac{dV}{dx} < 0$



05

$x = \frac{3P}{4}$ විට V උපරිම වේ. 05

එවිට AB දිග $= \frac{3P}{4}$ 05

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